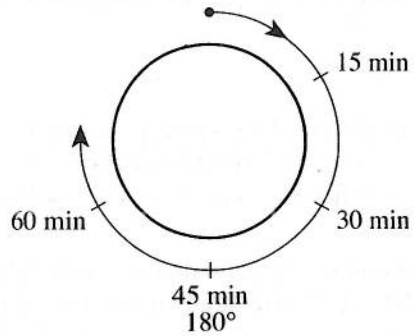
MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 1. The correct answer is C. The restaurant is rotating 180° in 45 minutes. That is 60° each 15 minutes. That is 240° every 60 minutes. You might want to sketch something like the drawing below.



If you like an algebraic solution, you can set up a proportion $\frac{180°}{45\text{min}} = \frac{x°}{60\text{min}}$ and solve it: $x = \left(\frac{180}{45}\right) = 4 \cdot 60 = 240$.

Most people who do not get this question right choose D, which gives the number of degrees the restaurant would rotate in 45 minutes if it made 1 complete rotation each hour.

Question 2. The correct answer is J. The 12 vases cost $18, so each vase costs $\frac{\$18}{12} = \$1.50$.

If you chose F, you probably divided 12 by 18 rather than 18 by 12. If vases cost $0.67 each, then 12 vases would cost less than $12.

Question 3. The correct answer is B. The longer side of the apartment is 30 feet long, and it is 6 inches long on the scale drawing. So, the length of the room, in feet, is 5 times the length on the drawing, in inches. Using this relationship, the length of the shorter side of the apartment is 5 times the 4 inches from the scale drawing. This is 20 feet.

Alternately, you could notice that the length of the shorter side is $\frac{2}{3}$ the length of the longer side on the drawing, and so the length of the shorter side of the room is $\frac{2}{3}$ of 30 feet, which is 20 feet.

These solutions are equivalent to using a proportion such as $\frac{6\text{ in}}{30\text{ ft}} = \frac{4\text{ in}}{x\text{ ft}}$ and solving: $x = \frac{4 \cdot 30}{6} = 4 \cdot 5 = 20$ feet.

Question 4. The correct answer is H. The total profit for the 5 years was, in millions, $8 + $8 + $8 + $9 + $9 = $42. Then the average profit, in millions, was $\frac{\$42}{5} = \$8.4$.

The most common wrong answer was J, which is the average of 8 and 9. Because there were more years with a profit of $8 million than with $9 million, the average for the 5 years must be closer to $8 million than to $9 million.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 5. The correct answer is A. If the van were driven for 20 miles, the cost for those miles would be $\$0.30 \cdot 20 = \$6$. Then the daily charge of \$25 would have to be added in, for a total of \$31. Similarly, if the van were driven for m miles, the cost for those miles would be $0.30m$ dollars, and the daily charge would make the total $0.30m + 25$ dollars.

C comes from treating the 30 cents like it was 30 dollars.

Question 6. The correct answer is F. The sum of the measures of all four interior angles in any quadrilateral is always 360°. The given three angle measures add up to $65° + 100° + 75° = 240°$, so the missing angle measure is $360° - 240° = 120°$.

If $\overline{AB}$ were parallel to $\overline{CD}$, then the measure of $\angle B$ and the measure of $\angle C$ would add up to 180° and the answer would be G. But the problem does not say that those sides are parallel, and it turns out that they are not parallel.

If you chose K, you may have calculated the average of the three given angle measures or taken the supplement of $\angle A$.

Question 7. The correct answer is C. The shorter two sides are the same length in $\triangle ABC$, so the same thing has to happen in any triangle similar to it. That means that $\overline{DE}$ is the same length as $\overline{EF}$, which is 3 meters. Then, the perimeter of $\triangle DEF$ is $3 + 3 + 5 = 11$ meters.

The most common incorrect answer is A, correctly finding the length of $\overline{DE}$ and stopping there.

Question 8. The correct answer is K. This problem can be solved by substituting the Celsius temperature ($C = 38$) into the formula and solving for F. The substitution step gives $F = \frac{9}{5}(38) + 32$, which can be solved as follows: $F = 68.4 + 32 \Rightarrow F = 100.4$. It is appropriate to round this to the nearest degree Fahrenheit because the precision of the Celsius temperature was only to the nearest degree Celsius.

If you chose J, you may have added $38 + 32$ and missed the $\frac{9}{5}$. An answer of H might come from calculating $\frac{9}{5}(38)$ correctly and forgetting to add in 32.

Question 9. The correct answer is B. Nick can only order whole cases, which contain 24 boxes of pens with 10 pens per box, for a total of $24 \cdot 10 = 240$ pens per case. An order of 2 cases would be 480 pens, which falls short of the desired 500 pens. To get 500 pens from his supplier, Nick needs to order 3 cases, and he will get 720 pens.

If you got answer A, you may have correctly divided 500 by 240 to get approximately 2.08 cases, but you may have rounded that to the nearest integer, which does not give the correct answer in this context. Answer E represents the number of boxes (not cases) of pens needed if Nick could order any whole number of boxes.

MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 10. The correct answer is K. When $a + b = 6$, then $2(a + b) + \frac{a+b}{6} + (a + b)^2 - 2$ becomes $2(6) + \frac{6}{6} + (6)^2 - 2$, which simplifies to $12 + 1 + 36 - 2 = 47$.

Question 11. The correct answer is C. If you bought 1 hamburger and 1 soft drink, it would cost \$2.10. If you bought 1 hamburger more, your order would cost \$3.50. So, the cost of the additional hamburger was \$3.50 – \$2.10 = \$1.40. Because 1 hamburger and 1 soft drink cost \$2.10, a soft drink must cost \$2.10 – \$1.40 = \$0.70.

Alternatively, you could set up two equations with two unknowns. Let h dollars be the cost of each hamburger and s dollars be the cost of each soft drink. Then $h + s = 2.10$ and $2h + s = 3.50$. Subtraction gives:

$$\begin{array}{rl} 2h + s & = 3.50 \\ \underline{-(h + s} & \underline{= 2.10)} \\ h & = 1.40 \end{array}$$

And then, substituting 1.40 for h in $h + s = 2.10$ gives $s = 2.10 - 1.40 = 0.70$.

The most common wrong answer is E, which is the correct cost of a hamburger. However, the question asks for the cost of a soft drink. Answer choice D is half of \$2.10, which would only be correct if a soft drink cost the same as a hamburger.

Question 12. The correct answer is K. There are many ways to solve this equation. One solution is the following.

$$\begin{array}{rrcl} \text{start with} & 12x & = & -8(10 - x) \\ \text{divide both sides by } -8 \;\Rightarrow & -\frac{3}{2}x & = & 10 - x \\ \text{add } x \text{ to both sides} \;\Rightarrow & -\frac{1}{2}x & = & 10 \\ \text{multiply both sides by } -2 \;\Rightarrow & x & = & -20 \end{array}$$

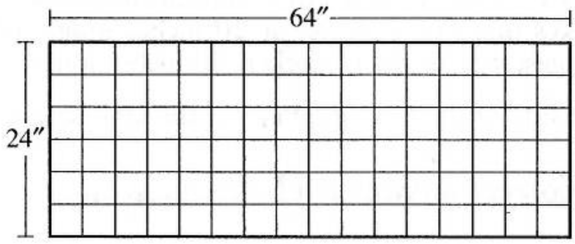
Another solution method would be to graph $y = 12x$ and $y = -8(10 - x)$ and see where the two graphs intersect. A calculator would produce these graphs, and you could find an approximate solution. That is good enough for this problem, because the answer choices are spread apart.

This is a problem where checking your answer is easy and can pay off. When $x = -20$, the left side of the original equation is $12(-20)$, which is -240. The right side is $-8(10 - (-20))$, which simplifies to $-8(10 + 20)$, then to $-8(30)$, then to -240. This solution checks. No other answer choice would satisfy the equation.

If you chose **H** or **J**, you may have multiplied out $-8(10 - x)$ to get $-80 + x$ or $-80 - x$ and done the rest of the steps correctly. You could have caught this by checking your answer. If you made an error with minus signs, you may have gotten one of the other answer choices.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 13. The correct answer is C. There would be 6 tiles along the 24″ side (6 · 4″ = 24″). There would be 16 tiles along the 64″ side (16 · 4″ = 64″). Then, 6 · 16 = 96 tiles are needed to completely cover the rectangular countertop:



An alternate solution is to figure the area of the countertop in square inches, 24 · 64, which is 1,536 square inches. Then, divide that by the area of a tile, which is 16 square inches. The result is 96, which is the number of tiles needed. The tiles cover the area without being cut because the side lengths of the countertop are divisible by the side length of a tile.

The most common wrong answer is E, which comes from correctly calculating the area of the countertop (1,536 square inches), but dividing by the length of the side of the square (4 inches) rather than by 4 · 4 square inches. If you chose A or B, you may have confused perimeter and area.

Question 14. The correct answer is H. If the answer choices give 2 of the 3 interior angle measures in a triangle, then the third angle measure is 180° minus the sum of the given angle measures. The chart below shows this calculation.

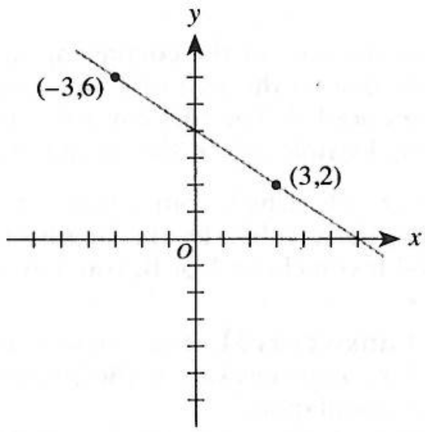
	1st angle	2nd angle	Sum of 1st + 2nd angles	3rd angle (180° – sum of 1st + 2nd angles)
F.	20°	40°	60°	120°
G.	30°	60°	90°	90°
H.	40°	100°	140°	40°
J.	45°	120°	165°	15°
K.	50°	60°	110°	70°

The only place where the third angle has measure equal to one of the first two angles is in H, where there are two 40° angles.

Question 15. The correct answer is C. The perimeter of the triangle, 66 inches, is the length of the three sides added together. Because one side is 16 inches long, the lengths of the other two sides added together must be $66 - 16 = 50$ inches. The ratio of the lengths of these two sides is 2:3. This ratio denotes 2 parts for the first side, 3 parts for the second side, and therefore 5 parts altogether. Because there are 50 inches altogether, and this must make up the 5 parts, each part is 10 inches long. That makes one side 2 parts, or 20 inches long, and the other side 3 parts, or 30 inches long. So the longest side of the triangle is 30 inches long.

B is the length of another side of the triangle, but not the longest side.

Question 16. The correct answer is H. Sketching a picture can be helpful.



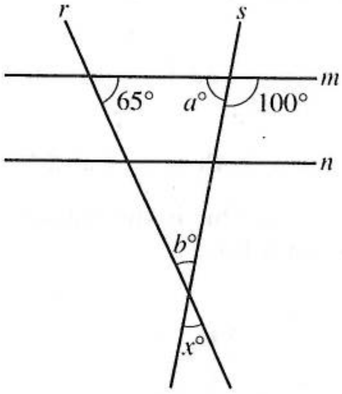
The points $(-3,6)$ and $(3,2)$ are on opposite sides of the y-axis, and an equal distance away from the y-axis, because the x-coordinates are 3 and –3. So, the midpoint will be the y-intercept of the line. The y-coordinate of the midpoint is $\frac{6+2}{2}$, which is 4.

If the points were not so nicely spaced with the y-axis, you could find the equation of the line through the two points and use it to identify the y-intercept. The slope of the line would be the change in y divided by the change in x, $\frac{6-2}{-3-3} = -\frac{2}{3}$. The point-slope form of the equation is then $y - 2 = -\frac{2}{3}(x - 3)$. To find the y-intercept, set $x = 0$ and solve for y, so $y - 2 = 2 \Rightarrow y = 4$. This shows that 4 is the y-intercept.

From the picture alone, you could deduce that the line had to cross the y-axis between 2 and 6. This observation eliminates all the answer choices except the correct one.

MATHEMATICS ■ PRACTICE TEST 2 ■ EXPLANATORY ANSWERS

Question 17. The correct answer is C. In the figure below, the angles with measure $a°$ and $100°$ form a straight angle along line m. This means $a° + 100° = 180°$, or $a° = 80°$. Now, you know two of the three angles in the larger triangle. The sum of all three must be $180°$. So, $80° + b° + 65° = 180°$, which means that $b° = 35°$. The angle measure $b°$ is equal to angle measure $x°$ because vertical angles have the same measure. So, $x° = 35°$.



If you chose A, you may have calculated $180° - (65° + 100°)$. These three given angle measures, $x°$, $65°$, and $100°$, do not need to add to $180°$. They are not the measures of the three interior angles in the *same* triangle.

Question 18. The correct answer is H. Each week, both ponds get shallower. The table below shows what happens for the first few weeks.

	Now	1 week	2 weeks	3 weeks	4 weeks
First pond	180 cm	179 cm	178 cm	177 cm	176 cm
Second pond	160 cm	159.5 cm	159 cm	158.5 cm	158 cm
Difference	20 cm	19.5 cm	19 cm	18.5 cm	18 cm

The ponds are getting closer and closer to the same depth, at the rate of 0.5 cm per week. Because the ponds started out 20 cm different, it will take $20 \div 0.5 = 40$ weeks to bring them to the same depth.

You could also check the answer choices. Answer F is not correct because, after 10 weeks, the first pond would be $180 - 10 = 170$ cm deep and the second pond would be $160 - 5 = 155$ cm deep. The other incorrect answers can be eliminated in the same way.

The most common incorrect answer is G. If you chose that answer, you may have reasoned that it would take 20 weeks for the first pond to get down to the level of the second pond. And that is correct, except that the second pond has gotten shallower during that 20 weeks, so the ponds are not the same depth.

Question 19. The correct answer is E. A line will always have an equation of the form $x = a$ or $y = mx + b$, for suitable constants a, m, and b. And, if a graph has an equation that can be put into one of these forms, the graph is a line.

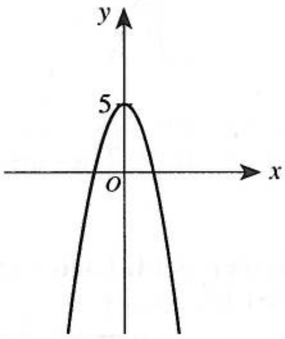
Equation A is already in the first form, where $a = 4$.

Starting with Equation B, divide both sides by 3 and you will get $y = 2$. This is the second form, with $m = 0$ and $b = 2$.

Equation C can be manipulated into the second form as follows: $x - y = 1 \Rightarrow -y = 1 - x \Rightarrow +y = -1 + x \Rightarrow y = x - 1$, which has $m = 1$ and $b = -1$.

Equation D is already in the second form, with $m = \frac{3}{4}$ and $b = -2$.

Equation E can be written as $y = -x^2 + 5$. This is the equation of a parabola, shown below. It is the only one of the equations that is not a line.



Question 20. The correct answer is G. All the answer choices are in terms of $\angle A$. Many people remember the trigonometric functions in the context of a right triangle, in terms of the lengths of the side opposite the angle, the side adjacent to the angle, and the hypotenuse. For the triangle given in the problem, the side opposite $\angle A$ has length 12 cm, the side adjacent to $\angle A$ has length 5 cm, and the hypotenuse has length 13 cm. The values of the trig functions are $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{13}$, $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13}$, and $\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$.

The only answer choice that gives one of these is G.

If you chose F, you may have been expecting the angle to be at one end of the horizontal base of the triangle. The value of $\cos B$ is $\frac{12}{13}$. Triangles can be rotated to any position. The terms "opposite," "adjacent," and "hypotenuse" are chosen to apply when the triangle is in an arbitrary position.

MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 21. The correct answer is B. Parallel lines have the same slope, so any line parallel to this line has the same slope as this line. To find the slope of this line, you could put it into slope-intercept form, and then the slope is the coefficient of x. $7x + 9y = 6 \Rightarrow 9y = -7x + 6 \Rightarrow y = -\frac{7}{9x} + \frac{6}{9}$, and so the slope is $-\frac{7}{9}$.

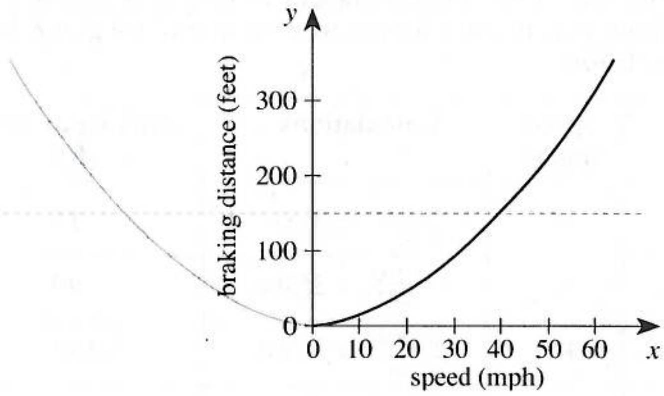
If you chose E, you probably knew that the slope is the coefficient of x, but that only is true when the equation is in slope-intercept form.

If you chose A, you may have gotten the equation into the form $9y = -7x + 6$ and then read off the coefficient of x. You didn't quite have it in slope-intercept form.

If you chose C, you may have made a mistake putting the equation in slope-intercept form.

Answer choice D could be reading off the constant 6 from the original equation.

Question 22. The correct answer is H. While it may not be worth your time to sketch a graph during the ACT, you should at least have a general idea of the situation in your mind. This is the equation of a parabola. Common sense will tell you that if the car is going faster, it will take a longer distance to brake to a stop. So, the parabola is opening upward.



The horizontal dashed line on the graph is where the braking distance is 150 feet. There are two points where the parabola intersects this line. One is to the left of the y-axis, which represents a negative speed for the car. If you solve an equation to find the intersection points, you will have to discard the intersection point with a negative speed.

If you took the time to create a graph, perhaps even on a graphing calculator, you might be able to estimate closely enough to choose among the answer choices. The desired speed is the x-coordinate of the intersection point.

To get this speed algebraically, the desired braking distance is 150 feet, and the equation $y = \frac{3(x^2 + 10x)}{40}$ gives the relation between speed (x) and braking distance (y). So you can solve the equa-

tion $150 = \frac{3(x^2 + 10x)}{40}$. One solution path starts by multiplying both sides by 40 and continues as follows: $150 \cdot 40 = 3(x^2 + 10x) \Rightarrow \frac{6{,}000}{3} = x^2 + 10x \Rightarrow x^2 + 10x - 2{,}000 = 0 \Rightarrow (x + 50)(x - 40) = 0 \Rightarrow x = -50$ or $x = 40$. You must discard the first solution. The only remaining solution is $x = 40$, which represents a speed of 40 miles per hour.

If you chose **J**, you may have done everything correctly except that when you solved $(x + 50)(x - 40) = 0$, you thought $x = 50$ was a solution. If you substitute 50 for x in the equation $(x + 50)(x - 40) = 0$, you will see that it is not a solution.

If you chose **F**, you may have made a numerical mistake when you substituted $x = 10$ into the braking-distance equation. The value is $\frac{3(10^2 + 10 \cdot 10)}{40} = \frac{3(200)}{40}$. It might be tempting to reduce $\frac{200}{40}$ to 50 and then get $3(50) = 150$, which is what you are looking for. But $\frac{200}{40}$ is 5, not 50.

Substituting the answer choices into the equation is a reasonable strategy for this problem. You can use the results from one of the answer choices to help you choose the next one to substitute, since you probably know that it takes longer to stop if you are going faster. The results of these substitutions are given below.

	Speed (mph)	Calculations	Braking distance (ft)
F.	10	$\frac{3(200)}{40} = 3(5)$	15
G.	30	$\frac{3(1{,}200)}{40} = 3(30)$	90
H.	40	$\frac{3(2{,}000)}{40} = 3(50)$	150
J.	50	$\frac{3(3{,}000)}{40} = 3(75)$	225
K.	60	$\frac{3(4{,}200)}{40} = 3(105)$	315

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

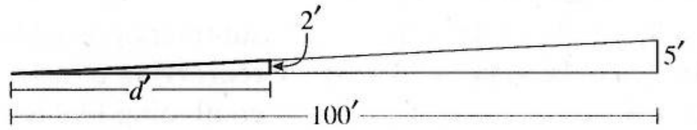
Question 23. The correct answer is A. Substitution gives $g(4) = \sqrt{4} = 2$ and $f(1) = 1^2 + 1 + 5 = 7$. Then $\frac{g(4)}{f(1)} = \frac{2}{7}$.

If you chose C, you may have started substituting 4 into function g and then continued substituting 4 into function f. The value of $\frac{g(4)}{f(4)}$ is $\frac{2}{25}$.

Question 24. The correct answer is K. There are 125 juniors who could be chosen. For *each* of those 125 juniors, there are 100 seniors who could be chosen. That makes $125 \cdot 100 = 12{,}500$ different pairs of students who could be chosen.

If you chose J, you probably added 125 and 100. If you chose G, perhaps you were figuring that once 100 pairs were formed, there would be no seniors left to put into another pair. Only 1 pair will be chosen, but there are many more than 100 pairs that are possible choices.

Question 25. The correct answer is C. The figure below shows the ramp.



The slope is given as rising 5 feet for every 100 feet of horizontal run. The ramp's rise is 2 feet, and the horizontal run is unknown. Let the horizontal run be represented as d. Then there is a proportion $\frac{2}{d} = \frac{5}{100}$. Its solution is $d = \frac{2 \cdot 100}{5} = 40$ feet.

If you chose A, you might have simplified 5% to 0.5. Then the proportion $\frac{2}{d} = 0.5$ has the solution 4 feet.

The most popular incorrect choice was B, which happens when the run is 5 times the rise. This gives a reasonable-looking ramp. However, the slope of such a ramp is $\frac{1}{5}$. The required slope is $\frac{1}{20}$.

Question 26. The correct answer is F. You could certainly test all of the answer choices to solve this problem. The first choice, −10, leaves the left side of the relation as $|-10 - 24|$, which simplifies to $|-34|$, which is 34. And $-34 \leq 30$ is false, so this is the correct answer.

Another way to solve this problem is to think about the interpretation of absolute value as a distance: $|t - 24| \leq 30$ means that the distance on the number line between t and 24 is at most 30 units. This distance would be greater than 30 only when t was more than 30 above 24 or more than 30 below 24. That is when t is more than 54 or t is less than −6. Answer choices G–K are all closer to 24 than this.

The most common wrong answer is 54 (K), which is right at the limit of how far away it can get from 24.

Practice ACT Tests

Question 27. The correct answer is D. The phrase "5 times a number n" can be represented as $5n$. If this is subtracted from 15, the expression $15 - 5n$ represents the result. Saying that this result is negative can be represented as $15 - 5n < 0$. Subtracting 15 from both sides gives $-5n < -15$, and then dividing both sides by -5 gives $n > 3$ (remember to reverse the direction of the inequality).

Most people who missed this chose E. This could come from not reversing the inequality in the last step of the solution, or it could come from subtracting 15 from $5n$ rather than subtracting $5n$ from 15.

Answer choice B is the result of solving $15 - 5n = 0$. This would give the boundary where the expression changes from positive to negative, so it's closely related to the problem you were asked to solve. If you chose this, you would still need to find the values of n where the expression is negative, knowing that $n = 3$ is the only place the expression is zero.

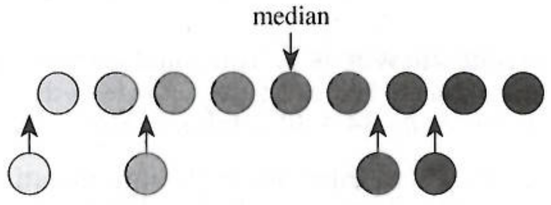
Question 28. The correct answer is J. A quick scan of the answer choices should give you a clue that combining like terms is in order.

$(x^2 - 4x + 3) - (3x^2 - 4x - 3)$	
$= (x^2 - 4x + 3) + (-3x^2 + 4x + 3)$	subtracting is adding the opposite
$= (x^2 + (-3x^2)) + (-4x + 4x) + (3 + 3)$	reordering terms
$= -2x^2 + 0x + 6$	combining like terms

If you chose K, you probably subtracted the $3x^2$ but *added* the $-4x$ and the -3. The solution above took care of this explicitly on the second line.

The other incorrect answers result from various combinations of errors with minus signs.

Question 29. The correct answer is B. Imagine the original 9 data items in order from smallest to largest. You might sketch a picture something like that shown below. The median is the 5th item in this list, because it is the middle value in the set.



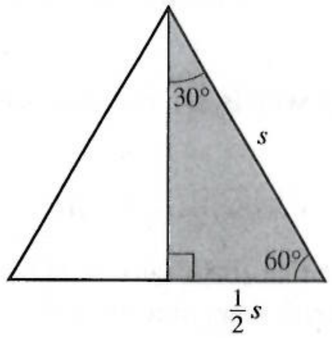
When the additional 4 data items are put into the list, there are $9 + 4 = 13$ items on the list. Because 2 of these items are greater than the original median, and 2 of these items are less than the original median, the original median is the middle value in the new set. That makes the original median the new median.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 30. The correct answer is K. The shaded area is the area of the larger circle minus the area of the smaller circle. The area of the larger circle is $\pi r^2 = \pi(10)^2 = 100\pi$ square centimeters. The area of the smaller circle is $\pi(5)^2 = 25\pi$ square centimeters. The difference is 75π square centimeters.

If you chose J, you likely found the difference in the perimeters of the two circles. You may have used a perimeter formula when you wanted an area formula. (Another possibility is that you calculated 10^2 as $2 \cdot 10$ and 5^2 as $2 \cdot 5$. But, 10^2 is $10 \cdot 10$ and 5^2 is $5 \cdot 5$.)

Question 31. The correct answer is E. The side lengths of a 30°-60°-90° triangle are in the ratio $1:\sqrt{3}:2$. If you didn't remember this, you could view a 30°-60°-90° triangle as half of an equilateral triangle, as shown below.



For the shaded triangle, you know that the base is half as long as the hypotenuse, because the base of the equilateral triangle is the same length as the other sides of the equilateral triangle. If the hypotenuse is s units long and the base is $\frac{1}{2}s$, then the Pythagorean theorem gives the height as

$$\sqrt{s^2 - \left(\frac{s}{2}\right)^2} = \sqrt{s^2 - \frac{s^2}{2^2}} = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \sqrt{\frac{3s^2}{4}} = \sqrt{3\frac{s^2}{4}} = \sqrt{3}\sqrt{\frac{s^2}{4}} = \sqrt{3}\frac{s}{2} = \frac{\sqrt{3}}{2}s.$$

This shows that the ratios are $\frac{1}{2}:\frac{\sqrt{3}}{2}:1$, which are equivalent to those given above, $1:\sqrt{3}:2$. (Obviously, it's quicker to know the ratios than to try to derive them each time you need them, but don't give up if you can't remember something—try to find it a different way.)

You could also use trigonometry to derive the side length ratios in a 30°-60°-90° triangle.

If you chose B, you might have been thinking of a 45°-45°-90° triangle, which has this ratio of side lengths.

If you chose D, you may have reasoned that because the angle measures are in the ratio 1:2:3, maybe the side lengths are in the ratio $\sqrt{1}:\sqrt{2}:\sqrt{3}$. This is a right triangle (it satisfies the Pythagorean theorem), but the angles are closer to 35°-55°-90°.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

The triangles in A and C are not right triangles. You could have eliminated them because they do not satisfy the Pythagorean theorem. For A, $\sqrt{1^2 + 1^2} = \sqrt{2} \neq 1$. For C, $\sqrt{1^2 + \left(\sqrt{2}\right)^2} = \sqrt{1 + 2} \neq \sqrt{2}$.

Question 32. The correct answer is G. If the x-coordinate is 20, then the y-coordinate can be found by substituting 20 for x: $0.005(20)^2 - 2(20) + 200 = 0.005(400) - 40 + 200 = 0.5(4) + 160 = 2 + 160$.

In theory, you could read the value off the graph, but you would not be able to read it accurately enough.

Many incorrect answers are caused by mistakes with the first term, $0.005x^2$. If you chose F, you may have made a decimal error calculating $0.005(20)^2$ and gotten 0.2 rather than 2.0. Other errors in calculation lead to **H**, **J**, and **K**.

Question 33. The correct answer is D. The distance formula (or the Pythagorean theorem) gives this distance directly. It is

$$\sqrt{(200 - 0)^2 + (0 - 200)^2} = \sqrt{200^2 + 200^2} = \sqrt{2 \cdot 200^2} = 200\sqrt{2} \approx 200(1.414) = 282.8$$

Another way to solve this is to notice that the length of $\overline{FO}$ is 200 units, and $\overline{FG}$ is longer. Also, the path from F to O to G is 400 units long, and the direct path along $\overline{FG}$ is shorter than this path. So, D is the only reasonable answer among those given.

Question 34. The correct answer is F. The shaded region is entirely contained in the given triangle because the curve is below the hypotenuse of the triangle, $\overline{FG}$. The area of the triangle is made up of the shaded area plus the unshaded area above the curve and inside the triangle. So, the shaded area is less than the area of the triangle.

Question 35. The correct answer is B. With $a = 4.2$, $b = 5.0$, and a 5° measure for $\angle C$, then the law of cosines gives the length of the third side of the triangle (the distance between the cargo ship and the fishing boat) as:

$$\sqrt{(4.2)^2 + (5.0)^2 - 2 \cdot 4.2 \cdot 5.0 \cos 5°}$$

If you chose C, you probably missed the minus sign in the middle of the expression. About half the students who do not get the correct answer choose C.

The 85° angle in answer choices D and E would be the angle measure by the fishing boat, but only if the angle by the cargo ship was a right angle. It turns out that it isn't a right angle.

MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 36. The correct answer is F. Because $a^3 = b$, then $b^2 = (a^3)^2$. Substituting this into the equation $c = b^2$ gives $c = (a^3)^2$. Because $(a^3)^2 = a^6$, the result is $c = a^6$.

The most common incorrect answer is G. If you chose that answer, you probably wrote $(a^3)^2$ as a^5. If you write $(a^3)^2$ out as $(a^3)(a^3)$ and then $(a \cdot a \cdot a)(a \cdot a \cdot a)$, you can see that it is a^6.

If you chose H, perhaps you wrote $(a^3)^2$ as $2(a^3)$.

Question 37. The correct answer is E. This sequence decreases by 17 – 12 = 5 each term. Here are some more terms:

Term #	1	2	3	4	5	6	7	8	9
Value	17	12	7	2	–3	–8	–13	–18	–23

Each of A–D can be verified from the chart. The common difference for D is defined so that it is positive if the sequence is increasing and negative if the sequence is decreasing. In symbols, if the sequence is represented by terms $a_1, a_2, a_3, \cdots$, then the difference between two terms is $a_{i+1} - a_i$ for all i. If this difference is constant for all i, then it is called the *common difference* and the sequence is called *arithmetic*.

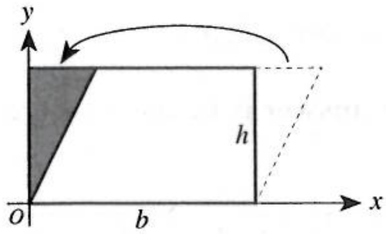
That leaves only E that could be false. The ratio of consecutive terms is defined by $\frac{a_{i+1}}{a_i}$ for all i. For the first two terms, the ratio is $\frac{12}{17}$, which is a bit more than 0.70. For the second and third terms, the ratio is $\frac{7}{12}$, which is a bit less than 0.59. That means the ratios are not equal for all terms, and so there is no common ratio. That means E is false.

The most common incorrect answer is B. If you chose this, perhaps you reasoned that, because the sum of the first 4 terms is 38, the sum of the first 5 terms cannot be anything less. Writing out more terms makes it clear that this can be so. Or, you may have made an arithmetic mistake finding the fifth term and so arrived at the wrong sum.

If you chose C, and had the correct sum for the first 5 terms, then perhaps you made an arithmetic mistake subtracting 5 for each term, or you miscounted terms. If you have a different sum for B and also have a different eighth term than C, you know that there is something amiss in your work and you should go back and try to find your mistake if there is time.

If you chose D, perhaps you did not understand the concepts of common difference and common ratio.

Question 38. The correct answer is G. The area of a parallelogram is given by bh, where b is the length of the base and h is the height (here, the distance between the bottom and the top). It sometimes helps to picture this formula geometrically.



If you cut off the right triangle from the right side, it fits exactly onto the left side to form a rectangle. The rectangle has the same area as the parallelogram. The rectangle's area is bh, where b is the length of the base and h is the height. This is even the same formula as for the parallelogram. The difference is that h is the length of a side of the rectangle, but it is not the length of a side of the parallelogram.

And so, for either the rectangle or the parallelogram, $b = 10$ coordinate units and $h = 6$ coordinate units, making the area $bh = 10 \cdot 6 = 60$ square coordinate units.

The distance from (0,0) to (3,6) is $\sqrt{3^2 + 6^2} = 3\sqrt{1^2 + 2^2} = 3\sqrt{5}$ coordinate units. If you chose J, you probably multiplied this side length by the length of the base, 10 coordinate units. This is the stereotypical mistake when figuring the area of a parallelogram. The picture above shows why the height is the right thing to use, not the length of the side.

If you chose H, you may have calculated the length from (0,0) to (3,6) as $\sqrt{6^2 - 3^2} = 3\sqrt{2^2 - 1^2} = 3\sqrt{3}$ and multiplied by the length of the base.

Question 39. The correct answer is D. The normal amount of lead is 1.5×10^{-5} milligrams per liter. This can be written as 0.000015 milligrams per liter.

Today's level, 100 times the normal amount, is $100(1.5 \times 10^{-5}) = 1.5 \times 10^{-5} \times 10^{2} = 1.5 \times 10^{-5+2} = 1.5 \times 10^{-3}$ milligrams per liter. This can be written as 0.0015 milligrams per liter. This is larger than the normal amount.

If you chose **A**, you likely added –100 to the exponent. This answer is $0.00000000\cdots0000000015$ milligrams per liter, where there are 104 zeros between the decimal point and the "15." This is smaller than the normal amount.

If you chose **B**, possibly you multiplied the exponent 2 from 10^2 by the exponent –5 from the normal amount. This is 0.00000000015 milligrams per liter, which is smaller than the normal amount.

If you chose **C**, you could have subtracted the exponent 2 from the exponent –5 in the normal amount. This gives 0.00000015 milligrams per liter, which is smaller than the normal amount.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 40. The correct answer is G. If you don't see a way to approach this problem right off, a useful general technique is to look for some concrete numbers that illustrate the conditions. One thought is to start checking numbers from 1,000 on up to find a perfect square. Using a calculator would be a good idea. $\sqrt{1{,}000} \approx 31.62$, $\sqrt{1{,}001} \approx 31.64$, $\sqrt{1{,}002} \approx 31.65, \ldots$ clearly, this will take a while. A perfect square between 1,000 and 9,999 would have to have its square root between $\sqrt{1{,}000}$ and $\sqrt{9{,}999}$ because the square root function is an increasing function. This will help you find examples more quickly: $25^2 = 625$ is too low, $35^2 = 1{,}225$ is in the interval. The smallest perfect square in this interval is $32^2 = 1{,}024$. Because $100^2 = 10{,}000$, the largest perfect square in this interval has to be $99^2 = 9{,}801$. The square roots of all of these perfect squares are the integers from 32 up to 99. All these square roots have 2 digits.

A quicker way is to find $\sqrt{1{,}000} \approx 31.62$ and $\sqrt{9{,}999} \approx 99.99$, which is a range that contains all possible square roots of the perfect squares. Any integer in this range (from 32 to 99) has 2 digits.

The most common incorrect answer is K. If you chose this answer, perhaps you made a calculation error that led you to believe you'd found a perfect square between 1,000 and 9,999 whose square root had 3 digits. And then you correctly found one whose square root had 2 digits. Your logic was correct. Some students probably choose K because they do not know how to solve the problem. Looking for concrete numbers would be a good strategy if you have time.

Question 41. The correct answer is B. This problem is in the general form $(a + b)^2$, which is equivalent to $a^2 + 2ab + b^2$ by the following derivation.

$$(a + b)^2 = (a + b)(a + b) = a \cdot a + a \cdot b + b \cdot a + b \cdot b = a^2 + 2ab + b^2$$

If $a = \frac{1}{2}x$ and $b = -y$, then $(a + b)^2 = (\frac{1}{2}x - y)^2$. This is equivalent to $a^2 + 2ab + b^2 = (\frac{1}{2}x)^2 + 2(\frac{1}{2}x)(-y) + (-y)^2 = \frac{1}{4}x^2 - xy + y^2$.

If you chose A, you probably squared the first term and squared the second term. This is the stereotypical error, and it's something that college math teachers (and high school math teachers) want you to know not to do. Something that might help you remember is to have a concrete example: $(1 + 3)^2 = 4^2 = 16$, but $1^2 + 3^2 = 1 + 9 = 10$.

If you chose C, you probably did everything correctly except remembering to square the $\frac{1}{2}$ to get $\frac{1}{4}$.

MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 42. The correct answer is F. To calculate a matrix product, you go across each row in the first matrix and down each column in the second matrix. You multiply the terms from a row by the corresponding terms from the column, and you add all of those terms together for the row-column combination and put the sum in that row-column of the result matrix. You do this for each row-column combination.

That explanation is all correct, but pretty abstract. Your math text might have some examples that you can look over to make things more concrete.

In this case, there is only 1 element in each row of the first matrix, and 1 element in each column in the second matrix. The matrix product is:

$$\begin{bmatrix} a \\ 2a \\ 3a \end{bmatrix} [1 \; 0 \; -1] = \begin{bmatrix} a \cdot 1 & a \cdot 0 & a \cdot (-1) \\ 2a \cdot 1 & 2a \cdot 0 & 2a \cdot (-1) \\ 3a \cdot 1 & 3a \cdot 0 & 3a \cdot (-1) \end{bmatrix} = \begin{bmatrix} a & 0 & -a \\ 2a & 0 & -2a \\ 3a & 0 & -3a \end{bmatrix}$$

The computations in G are correct, but the terms were put in the wrong places.

If you chose K, you may have reasoned that whatever role the 1 played in the second matrix, the −1 would cancel it out, and the 0 would not change that. If all of the terms were combined, that would be the case. But a matrix can keep the terms separate.

Question 43. The correct answer is D. Because the base is a straight line, these two angle measures add up to 180°. In the language of algebra, this can be represented as $(4x + 6) + (2x) = 180$. Solving this for x gives $6x + 6 = 180$, then $x + 1 = 30$, then $x = 29$. This makes one angle measure $4(29) + 6 = 122$ degrees and the other angle measure $2(29) = 58$ degrees. (Check: Is 122° + 58° equal to 180°? Yes.)

The problem asks for the measure of the smallest of these two angles. That is 58°.

If you chose C, you probably just stopped as soon as you found the value of x. The problem asks for something different. The other popular incorrect answer is E.

Question 44. The correct answer is G. One way to solve this problem is to list out all the numbers in this range that you think might be prime, then check to see if any of them factor. You probably know that you don't have to check the even numbers. That leaves the following list:

31 33 35 37 39 41 43 45 47 49

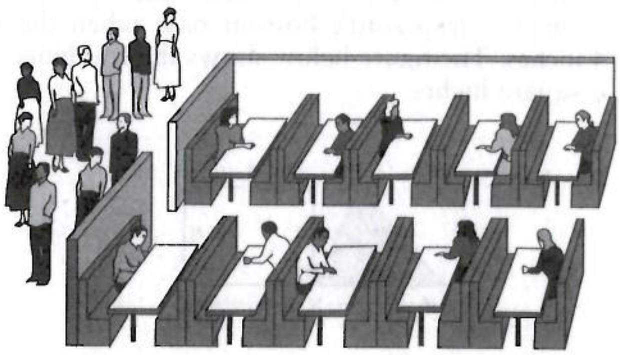
If one of these does factor, it will have a prime factor of at most $\sqrt{49}$, which is 7. You have already eliminated all multiples of 2. If you eliminate all multiples of 3, 5, and 7, anything left on the list is a prime number. The multiples of 3 on the list are 33, 39, and 45. You can eliminate 35 because it is a multiple of 5. You can eliminate 49 because it is a multiple of 7. Then all the numbers remaining, namely 31, 37, 41, 43, and 47, are prime numbers.

If you chose H, you must have counted a number as prime that really isn't prime. You might want to figure out which one that was. People who miss this problem tend to count extra numbers as primes.

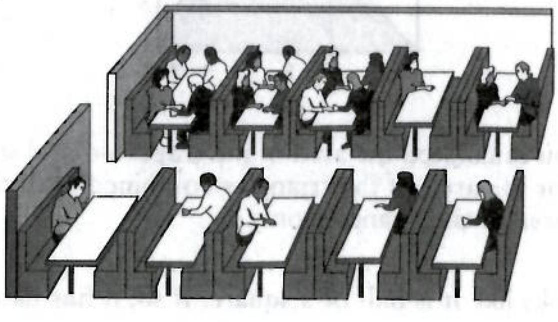
MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 45. The correct answer is E. The cotangent of $\angle A$ in this right triangle is the length of the leg adjacent to the angle divided by the length of the leg opposite the angle. That ratio is $\frac{\sqrt{4-x^2}}{x}$. If you chose D, you chose the tangent of the angle. Answer choice C represents the sine of the angle. Answer choice B is the cosecant of the angle. If you want to get problems like this correct, you need to have a way to keep the trig functions straight.

Question 46. The correct answer is J.



Because no booths can be empty, imagine 1 person sitting in each booth. That leaves 10 people standing around waiting for you to tell them where to sit. How many booths can you fill up with another 3 people each? Well, you can get 3 groups of 3 people from the 10 who are still standing, with 1 person left over. That means that you can fill at most 3 booths with 4 people. There will be 1 booth with 2 people, and the other 6 booths will have 1 person. (Check: Are there 20 people? 3(4) + 0(3) + 1(2) + 6(1) = 12 + 2 + 6 = 20. Yes. Are there 10 booths? 3 + 0 + 1 + 6 = 10. Yes.)



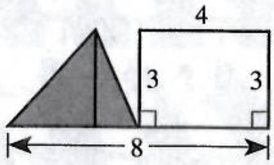
If you chose K, you filled up 5 booths with 4 people each. But, you left the other 5 booths with no one sitting there. The problem specified that NO booths are empty. (This is the most common wrong answer.)

If you chose H, you may have just miscounted. Make sure you check your work. You might want to draw a diagram for problems like this so that you can check your work easily.

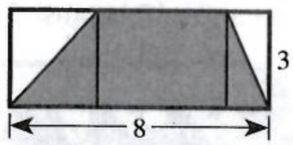
MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 47. **The correct answer is B.** The area of the trapezoid is $\frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(8 + 4)3 = 6 \cdot 3 = 18$ square inches. The area of the unshaded rectangle is $4 \cdot 3 = 12$ square inches. The triangles and the rectangle together form the trapezoid, so the area of the trapezoid minus the area of the rectangle is the area of the triangles. And this area is $18 - 12 = 6$ square inches.

Another way to solve this problem is to slide the two shaded triangles together and calculate the area of the new triangle. From the original figure, notice that the combined base of the triangles is the amount left over from the trapezoid's bottom base when the width of the rectangle is removed. That is $8 - 4 = 4$ inches. The figure below shows the combined triangles. Their combined area is $\frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$ square inches.



The most common incorrect answer is D, which could come up in a variety of ways. Many students have trouble finding the area of trapezoids and resort to calculating bh as if the trapezoid were a rectangle or parallelogram. The figure below shows the trapezoid inside a rectangle with base 8 inches and height 3 inches. The rectangle has area bh, which is clearly larger than the area of the trapezoid.



If you chose E, perhaps you calculated the area of the trapezoid and stopped. Perhaps you thought the problem was asking for the area of the triangles combined with the area of the rectangle, but it asks for the combined area of the triangles only.

The triangle on the left looks like it is half of a square. If so, it has base 3 inches and height 3 inches and then area $\frac{1}{2}(3)(3) = \frac{9}{2}$ square inches. If the other triangle has the same area, the combined area of the triangles is $\frac{9}{2} + \frac{9}{2} = 9$ square inches. Perhaps you reasoned this way if you chose C.

MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 48. The correct answer is G. Each of the corner triangles are right triangles because they share an angle with the square. Both legs of these right triangles are 6 inches long because they are half the length of the side of the square. So, the hypotenuse of each of these triangles is $\sqrt{6^2 + 6^2} = 6\sqrt{1^2 + 1^2} = 6\sqrt{2}$ inches. The perimeter of *EFGH* is made up of 4 of these hypotenuses, so the perimeter of *EFGH* is $4 \cdot 6\sqrt{2} = 24\sqrt{2}$ inches.

You could have used the ratio of sides in an isosceles right triangle, rather than the Pythagorean theorem, to get the hypotenuse of the corner triangles. The basic flow of the solution is the same.

If you chose F, perhaps you noticed that the area of *EFGH* is half the area of *ABCD*. That's a good observation. That does not mean, though, that the perimeter of *EFGH* is half the perimeter of *ABCD*. If that were true, then each side of *EFGH* would be 6 inches long, and the corner triangles would have 3 sides of length 6 inches. The triangle must then be equilateral and have a right angle. That can't happen. (The perimeter of a figure that is geometrically similar to the original and has area in the ratio 2:1 has perimeter in the ratio $\sqrt{2}$:1.)

Question 49. The correct answer is A. To see this, start with the statement $\sqrt{y^2} = |y|$, which is true for all real values of y. (If you wonder why there is an absolute value in this equation, test $y = -5$.) Substitute $-x$ for y in the equation. The result is $\sqrt{(-x)^2} = |-x|$, which is expression I.

On the other hand, II is always positive or zero, and III is always negative or zero, so the only time they are equal is when x is zero. For example, when $x = 1$, $|-1| = 1$ but $-|1| = -1$. So, II and III are not equivalent.

The most common incorrect answer is C. Even though expressions II and III look a lot alike—they have the exact same symbols in almost the same order—they are not equivalent. Expression II is never negative and III is never positive.

If you answered E, you likely saw that II and III are not equivalent. Expression I looks so much different than either of the others that it is tempting to just say it can't be the same. One approach would be to test a few numbers in the expressions. Be sure you test a positive number and a negative number. If you substitute these into I and into II correctly, you will find they match. Then, you should suspect that I and II could be equivalent and look for a way to convince yourself of this.

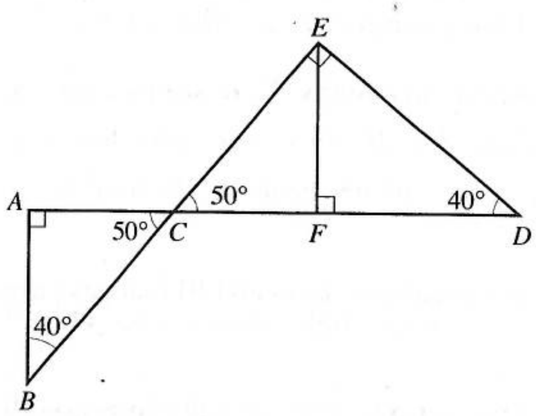
Question 50. The correct answer is K. Statement F, that $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ are parallel, is true because both are perpendicular to the same other line, $\overleftrightarrow{AD}$.

Because $\angle DEB$ is marked with a right angle, $\overline{DE}$ is perpendicular to $\overline{BE}$, and so G is true.

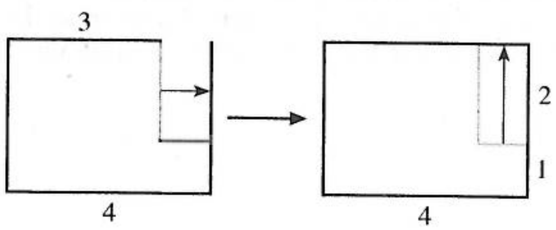
H is true because the two angles, $\angle ACB$ and $\angle FCE$, are vertical angles.

Statement J is true because the angles in $\triangle BAC$ are congruent to the angles in $\triangle EFC$. First, $\angle A$ is congruent to $\angle EFC$ because they are both right angles. Next, $\angle ACB$ and $\angle FCE$ are vertical angles. And third, the remaining angles have to have the same measure because the sum of the interior angle measures in any triangle is 180°.

Because all of the other statements are true, you should expect that statement K could be false. The following diagram shows such a case. (Only if $\angle ECF$ is congruent to $\angle EDF$ can $\overline{CE}$ be congruent to $\overline{DE}$.)



Question 51. The correct answer is D. This geometric figure has 6 sides, but you are only given the length of 4 of those sides. One of the slickest ways to find the perimeter is to move two of the sides to form a rectangle with the same perimeter. The drawing below shows the new rectangle.



The bottom of this rectangle is 4 inches long, and the right side is 3 inches long. So, the perimeter is $4 + 3 + 4 + 3 = 14$ inches.

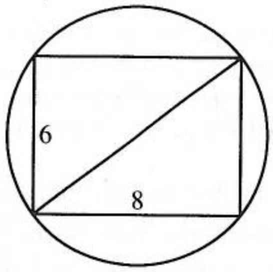
MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

An alternate method is to deduce that the left side is 3 inches long because it is the same length as the vertical sides on the right. And, the last unknown side is 1 inch long because it is the difference between the horizontal 4-inch side on the bottom and the horizontal 3-inch side on the top. Then, the perimeter (going clockwise) is $4 + 3 + 3 + 2 + 1 + 1 = 14$ inches.

If you chose C, you may have forgotten to add in the horizontal side that connects the vertical 2-inch side and the vertical 1-inch side.

If you chose A, you may have added just the numbers given directly on the figure. Or, you may have calculated the area instead of the perimeter.

Question 52. The correct answer is H. To find the area of the circle, all you need to know is the radius of the circle. If you drew in a diagonal of the rectangle, it would look something like the following.



The diagonal of the rectangle is a diameter of the circle. By the Pythagorean theorem (do you see the right triangle?) the length of the diagonal is $= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ inches. So, the radius is 5 inches. That makes the area $\pi(5)^2 = 25\pi$ square inches.

The most common incorrect answer is J, which you might get by finding the area of the rectangle and multiplying by π.

If you chose G, you may have thought that the diameter of the circle was 8 inches because that is the longest segment drawn on the picture. Then, the radius would be 4 inches and the area would be $\pi(4)^2 = 16\pi$ square inches. The diameter is 10 inches, which is longer than the 8-inch side, as the diagram above shows.

Question 53. The correct answer is D. Marshall made 24 calls on the first day. He makes 5 more calls each day than he had the day before. That means he made 29 calls on the second day. The table below shows the number of calls he made on each of the 20 days.

day	1	2	3	4	5	6	7	8	9	10
# calls	24	29	34	39	44	49	54	59	64	69

day	11	12	13	14	15	16	17	18	19	20
# calls	74	79	84	89	94	99	104	109	114	119

You could try to add all of these up. You might make a mistake, even with a calculator. But, that method is straightforward and would work.

Another approach is to notice that the number of calls for the first day and the last day add up to 143, as do the number of calls for Day 2 and Day 19, as do the number for Day 3 and Day 18, as do all the other pairs of days, working forward from the beginning and backwards from the end. There are 10 pairs of days, each with 143 calls. That is 1,430 calls for the 20 days.

A third approach relies on accurately remembering the following formula for the sum of an arithmetic series: $S_n = \frac{n}{2}[2a + (n - 1)d]$, where the first term in the series is a, the common difference between terms is d, there are n terms in the series, and the sum is S_n. Substituting the quantities you know gives $\frac{20}{2}[2 \cdot 24 + (20 - 1) \cdot 5] = 10[48 + 95] = 10[143] = 1{,}430$ calls.

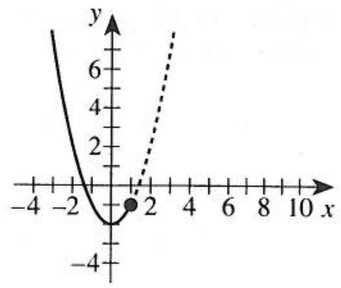
If you chose B, you may have calculated the number of calls for Day 10 and multiplied this by 10.

If you chose C, you may have broken the problem into 2 parts, the base 24 calls and the additional calls. This is a good strategy. The additional calls now are a geometric series $(5 + 10 + 15 + \cdots + 95)$. This has 19 terms (remember that on Day 1 there are no additional calls). The sum of the series is $19 \cdot \frac{5 + 95}{2} = 950$. Great. Then $950 + 24 = 974$ calls. Unfortunately, you've only added in the base 24 calls for 1 day. Marshall made the base 24 calls on all 20 days. You'd need to add $20 \cdot 24 = 480$ instead of just 24. This gives $950 + 480 = 1{,}430$ calls, which is correct.

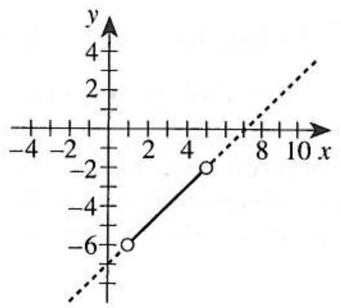
Question 54. The correct answer is K. This is called a *piecewise-defined function* because it is pieces of different functions put together to form a single function. When $x \le 1$, the equation is $f(x) = x^2 - 2$, so $f(1) = 1^2 - 2 = -1$ and $f(0) = 0^2 - 2 = -2$. (Actually, this information is enough to eliminate all the graphs except the correct one. But you might want to read the rest of this explanation anyhow.)

The graph of $y = x^2 - 2$ is a parabola, opening upward. Its graph is shown on the next page. The only part of the graph that applies for this piecewise defined function is the part where $x \le 1$. The rest of the graph is represented with dashes.

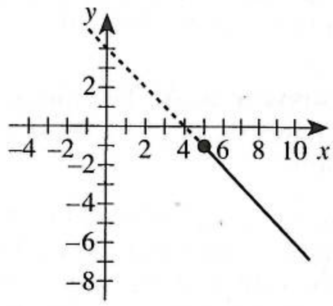
MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS



When $1 < x < 5$, then $f(x) = x - 7$, so $f(2) = 2 - 7 = -5$ and $f(4) = 4 - 7 = -3$. This is a straight line. A graph is shown below, with the parts outside $1 < x < 5$ shown with dashes and open circles.



When $x \geq 5$, the equation is $f(x) = 4 - x$, so $f(5) = 4 - 5 = -1$ and $f(6) = 4 - 6 = -2$. This is also a straight line. Its graph is shown below, with the part outside $x \geq 5$ shown with dashes.



K puts all of these pieces together.

MATHEMATICS • PRACTICE TEST 2 • EXPLANATORY ANSWERS

Question 55. The correct answer is D. The table below shows the value of the cosine function at values of θ that are the endpoints of the intervals from each of the answer choices.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
cos θ	1	$\frac{\sqrt{3}}{\pi}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1
approximation of cos θ	1	0.866	0.5	0	− 0.5	-1

The value −0.385 is between 0 and −0.5, which is saying that the value of cos θ is between cos $\frac{\pi}{2}$ and cos $\frac{2\pi}{3}$. Because cos θ is continuous, that means there is a value of θ between $\frac{\pi}{2}$ and $\frac{2\pi}{3}$ that satisfies the conditions of the problem.

If you chose C, you might have been looking for a place where cos θ = +0.385 rather than −0.385.

If you chose A or E, you might have been looking for a place where sin θ = 0.385.

Question 56. The correct answer is J. The equation $(x - 6a)(x + 3b) = 0$ has these two solutions. (You can check this by substituting $6a$ in for x and substituting $-3b$ in for x.) Multiplying out this equation gives $x^2 + (-6a + 3b)x + (-6a)(3b) = 0$, which is the same as $x^2 + (-6a + 3b)x - 18ab = 0$.

If you chose F, you may have gotten the initial equation right, $(x - 6a)(x + 3b) = 0$, but then multiplied incorrectly to get $(x)(x) + (-6a)(3a) = 0$.

Most of the other incorrect answers could be due to mistakes with negative signs. G comes from the initial equation $(x + 6a)(x - 3b) = 0$, where the signs are opposite what they should be. (If you substitute $6a$ in for x, you will not get zero on the left side of the equation.) If you chose H, you may have started with the equation $(x - 6a)(x - 3b) = 0$.

Question 57. The correct answer is A. The midpoints of the sides of the square are on the circle. These points have coordinates (0,3), (3,6), (6,3), and (3,0). The first point, (0,3), satisfies A, but none of the other equations.

Alternately, because the circle centered at (h,k) with radius r has the equation $(x - h)^2 + (y - k)^2 = r^2$, you can find the equation by finding the center and radius. From the diagram, you can see that the center of the circle is the same as the center of the square, which is (3,3). Also, the radius of the circle is the distance from (0,3) to (3,3), which is 3 coordinate units. So, $(x - 3)^2 + (y - 3)^2 = 3^2$ is an equation of the circle.

One common mistake is to remember the equation of the circle incorrectly, with plus signs where there should be minus signs (C). Another common mistake is to not square the radius on the right side of the equation (B). Or, some people do both of these things (E). If you chose D, you may have used the diameter on the right side of the equation, or you may have used the largest coordinate from the figure (and used plus signs instead of minus).

MATHEMATICS ▪ PRACTICE TEST 2 ▪ EXPLANATORY ANSWERS

Question 58. The correct answer is J. Let L_1 feet be the length of Pendulum 1 and t_1 seconds be the time for a complete swing of Pendulum 1. Let L_2 and t_2 describe Pendulum 2 in the same way. The time for a complete swing of Pendulum 1 is triple the time required for a complete swing of Pendulum 2. This means that $t_1 = 3t_2$.

The most common approaches to solving problems like this involve finding an equation that contains the two variables the question asks about, here L_1 and L_2, and solving for one variable or solving for the ratio of the variables.

By the equation that relates L to t, the equation $t_1 = 3t_2$ becomes $2\pi\sqrt{\frac{L_1}{32}} = 3 \cdot 2\pi\sqrt{\frac{L_2}{32}}$ (which is an equation that contains both L_1 and L_2). Dividing both sides by 2π gives the equation $\sqrt{\frac{L_1}{32}} = 3\sqrt{\frac{L_2}{32}}$. Squaring both sides gives $\frac{L_1}{32} = 9 \cdot \frac{L_2}{32}$. Multiplying both sides by 32 gives $L_1 = 9L_2$. So, Pendulum 1's string is 9 times the length of Pendulum 2's string.

The most common incorrect answer is G. If you chose that, you might have reasoned that if one variable triples, then any other variable must also triple. This happens for some functions, notably linear functions, but the function in this problem is not linear. The value of L must go up by a factor of 9 so that the square root will go up by a factor of 3.

If you chose H, you may have reasoned that, in order for the square root to go up by a factor of 3, the quantity under the square root must go up by a factor of 6. That would mean that L would go up by a factor of 6. This turns out not to be enough, because if L goes up by a factor of 6, the square root will only go up by a factor of $\sqrt{6}$, which is less than 3.

Question 59. The correct answer is A. By properties of logarithms, $\log_a (xy)^2 = 2 \log_a (xy) = 2(\log_a x + \log_a y) = 2(s + t)$.

If you chose D, you may have done all of the steps above correctly, except for thinking that $\log_a (xy) = \log_a x \cdot \log_a y$ (using multiplication rather than addition on the right side).

Answer C can come from thinking that $\log_a (xy)^2 = \log_a (x^2y^2) = \log_a x^2 \cdot \log_a y^2 = 2 \log_a x \cdot 2 \log_a y$. The first and last steps are correct, but not the middle one.

Question 60. The correct answer is F. Let d be Jennifer's distance in 1990. Her distance in 1991 would be $1.1 \cdot d$. And her distance in 1992 would be $1.2(1.1 \cdot d)$, which simplifies to $1.32 \cdot d$. This represents an increase of 32% from 1990 to 1992.

The most common incorrect answer is G. If you chose this, you probably added the 10% and the 20% to get 30%. This would work fine if the percents were percents of the same thing. But the first increase is a percent of the 1990 distance while the second is a percent of the 1991 distance.

One good strategy for investigating problems like this is to choose a specific number to represent the initial distance. Say you choose 10 feet for her 1990 distance. The 1991 distance would be the original 10 feet plus a 10% increase, which is 1 more foot, for a total of 11 feet. To find the 1992 distance, take the 11 feet and add 20% of this amount, which is 2.2 feet. Then, the 1992 distance is $11 + 2.2 = 13.2$ feet. The percent increase from the original 10 feet is $\left(\frac{13.2 - 10}{10}\right)(100\%) = (0.32)(100\%) = 32\%$. These are the exact same operations as were done in the first method, but the calculations are done with concrete numbers rather than abstract variables.